

## Microstrip Line on Ground Plane with Closely Spaced Perforations - Simple CAD Formulas by Synthetic Asymptote

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**Abstract-Simple CAD formulas, of a microstrip line on a ground plane with periodic perforations, closely spaced and at low frequency, are derived for possible application in the multi-layer circuits in an LTCC package. The derivation is done by the novel technique of synthetic asymptote. Compared with the hardware experiment, the average error of the CAD formulas can be as low as 2.5 %. The formulas have no arbitrary constants and give good physical insights.**

### 1. Introduction

Simple CAD formulas are derived for the characteristic impedance and propagation constant of a microstrip line with a ground plane of periodic perforations of period much less than one wavelength. This study should help latest circuit packaging technology of the multilayered LTCC (low temperature co-fired ceramic) [1]. There the perforation is needed for the passage of the vias from one circuit layer to the next.

Each circuit layer is quite small, of the order of say 1 cm square. The small size is to satisfy the even increasing demand of circuit miniaturization. In terms of the period of the ground plane perforation, say 1 mm or less, therefore, even 5 GHz. may be considered low frequency. This is the reason that the CAD formula in this paper is for low frequency, and not for the "photonic-band-gap" type of periodic ground plane" [2] at a higher frequency where resonance occurs.

At higher frequencies it would be more difficult to derive a CAD formula. On the other hand, it is difficult to do a numerical solution by the usual moment method, for the perforated ground, at whatever frequency [2].

### 2. Properties of Synthetic Asymptote

At low frequency, the approach of synthetic asymptote [3] for electrostatics may be used to derive the CAD formulas. Synthetic asymptote

is basically a curve fit between two regular asymptotes of a parameter of interest. A curve fitted formula, frequently works well for parameter values within the range of known data points, but diverges outside the range. This does not happen in synthetic asymptote fitting as there are not parameter values outside the limits of the asymptotes. As a result the synthetic asymptote usually give quite accurate solutions.

### 3. The Layout of the Microstrip Line with Perforated Ground Plane

The CAD formulas of the microstrip line is to be checked by hardware and software experiments.

The periodic perforations on the ground plane are done in a square grid. Also, the microstrip lines are assumed to run above and essentially along the non-perforated area of the ground plane. The layout is shown in Fig. 1.

If a microstrip line is chosen to run above the *perforated* area of the ground plane, The perforation means that the returning ground plane current cannot run along the areas directly below the microstrip line but along the more distant non-perforated area. This would give a high inductance and a high characteristic impedance. The high characteristic impedance may not be acceptable. As a result a formula for such arrangement is not done.

### 4. The CAD formulas by Synthetic Asymptote

The top view of the structure is given in Fig. 1. The CAD formulas are given below. The formulas are:

$$Z = \sqrt{\frac{L}{C}} \quad (1a)$$

$$\epsilon_{eff} = C / C_{air} \quad (1b)$$

and (with power  $n = 1.08 - 0.433 p^2$  used below, and  $p$  = fraction of metal surface removed from the ground plane)

$$C = \epsilon_0 \left\{ \left( \frac{\epsilon_r W_a}{h} \right)^n + (\epsilon_r + 1) \mathbf{P} \left[ \left( \frac{1}{\ln(\frac{8h}{W} + 1)} - \frac{W}{8h} \right) + \left( \left( \frac{W_s - W_a}{W_s} \right) \frac{1}{\ln(\frac{8h}{W_a} + 1)} - \frac{W_a}{8h} \right) e^{-2p\sqrt{\left(\frac{h}{W_s}\right)^2 - \left(\frac{h}{I}\right)^2}} \right]^n \right\}^{1/n} \quad (2)$$

Here  $C$  is used for both capacitance calculation and inductance  $L$  calculation through duality. For capacitance calculation, in (2), from overlapped parallel plate area observed in Fig. 1, we take  $W_a = W_c$  and  $W_c = W$  or  $W(1-h)$  whichever is smaller, and  $h = (W_s - W_g)(W - W_g)/WW_s$  is the fraction of strip facing the perforated area, i.e., un-overlapped area. The symbols  $W$  are defined in Fig. 1.

We can identify the first term to be the near asymptote (regular) of small substrate thickness  $h$  from a *parallel plate* consideration.

The second-plus-correction term (in a square bracket) is the far asymptote of large substrate thickness making the perforated ground plane to appear to be a solid plate [4]. This attached correction term is to ensure that the far asymptote does not dominate the near asymptote at the near limit of  $h$ . This second term is *the fringe field of the strip*.

The third-plus-correction term (in another square bracket) is the intermediate term between the near and far asymptotes. This latter attached correction term serves a similar purpose as the one before. The third term accounts for the periodic perforations on ground plane and is derived from space harmonic functions. These three terms together then is the synthetic asymptote for any substrate thickness and any ground plane perforation.

Because of the vector flow of the electric current over the *perforated* ground plane, the area of parallel plate overlap is less than that of the scalar charge. This means that the inductance  $L$  is not equal to  $\mathbf{m}_0 \epsilon_0 / C$  with  $\epsilon_r$  set to unity. Rather, it can be shown that it is the *inductance* duality of  $L = \mathbf{m}_0 \epsilon_0 / C_{air}$ , that is:  $C_{air} = C$  in (2) but with  $\epsilon_r$  set to

unity and also  $W_a = W$  or  $W_g$  whichever is smaller. Here  $W_g$  is smaller than the  $W(1-h)$  of the above for capacitance.

There is still a little leftover fringe field after the second and third terms in (2). This is

corrected by the power of  $n$  as defined above (2). This is the only one parameter in the formula that is numerically matched. It is matched at two data points from the software results of WIPL in the next section.

With  $p = 0$ , the ground plane is a solid sheet. The power  $n = 1.08$  then. This power agrees with the power of the same in [6] to 1%.

With both  $L$  and  $C$  found, the formula of the propagation constant is easily derived.

## 5. The Experiments—Hardware and Software

The hardware experiment is done in large ground plane of 10 cm square. The substrate thickness  $h$  is 1.6 mm with  $\epsilon_r = 4.0$ . In Fig. 1, the microstrip width  $W$  is 5 mm. The perforation period  $W_s$  between cells is 1 cm. The perforation in each cell deletes percentages ( $p \times 100\%$ ) of metal. In a series for measurement, they are: 0, 25, 36, 49, 64 and 81 %.

These give the remaining width of the strip  $W_g$  under the strip to be 10, 5, 4, 3, and 1 mm. The 0 % deletion (i.e.,  $W_g = 10$  mm and  $p = 0$ ) means no perforation.

While the perforation period is 1 cm, the perforated ground plane size is 10 cm. The derived formula, on the other hand, assumes a ground plane of infinite size. To avoid the error of possible fringe field effects on the ground plane of such size with surroundings, the hardware measurements are done at a low frequency of 50 MHz.

If ground plane of effectively infinite size were used, the measurements can be done up to 500 MHz before the 1 cm period (at  $\epsilon_r = 4.0$ ) gets larger than the static criterion of  $I/20$ . In LTCC, the period can be 1 mm, or less. This means that the static formulas derived below are applicable to 5 GHz, and may be beyond.

The numerical experiment here uses the software WIPL [5]. The physical dimensions are the same as those of the hardware experiment except for two changes, the ground plane size and frequency. A smaller ground plane of 3x5 cm is used to reduce the computing time needed as the perforation necessitates the inclusion of the ground plane in computation. The frequency used in the computation is not 50 MHz but higher at 350 MHz, because of the low frequency limitation of the software. These two changes should not cause much error as mentioned in the last paragraph that the upper frequency limit is 500MHz for the perforation period used. Also, the substrate is only 1.6 mm thick. That is: the substrate is very thin in wavelength, and gives very short range in induction field to ever reach the edge of the ground plane.

As shown in Fig. 2 below, the agreement between numerical and hardware experiments is excellent, with only 1 to 2% deviation.

## 6. Comparison of Results of the Formulas and Experiments

Three results, from the CAD formulas of (2), from the hardware and software experiments, are given in Fig. 2. The abscissa is the percentage of perforation, i.e.,  $p \times 100\%$ . As observed in Fig. 1,  $p$  is given by  $p = (W_s - W_g)^2 / W_s^2$ . We see in Fig. 2 that the agreements in  $Z_0$ , the characteristic impedance, between the results are surprisingly good, with maximum error of less than 5%, or the average error of, say, 2.5 %.

Good agreements of this sort give us confidence in the formula (2) and the synthetic asymptote approach. We have found that the errors in  $\epsilon_{eff}$ , the effective dielectric constant, is of similar to that of  $Z_0$ .

The reason for such good agreements is as follows. In addition to the near and far field effects (parallel plate and strip fringe) in (2), we also include the intermediate field effect, that is: the ground plane fringe field due to the perforation.

## 7. Conclusion

The substrate thickness is kept at  $h = 1.6$  mm that is about 1/3 of the strip width. This means in formula (2) that the strip fringe field (2<sup>nd</sup> term) contribution becomes comparable with the parallel plate (1<sup>st</sup> term) contribution. The

strip fringe field (2<sup>nd</sup> term) assumes that the ground plane is not perforated but a solid sheet. Then the terms of the strip fringe field and the parallel plate together means that the ground perforation is effectively reduced even if the small ground plane fringe (3<sup>rd</sup> term) remains unchanged. With this  $h$  value therefore, in Fig. 2 the changes in  $Z_0$  with different perforation are not large, i.e.,  $Z_0$  values are close to that of a solid ground plane.

With a thinner substrate  $h$  and narrower  $W_g$  under the strip  $W$ , the near asymptote of the parallel plate effect in  $C$  and  $L$  in (2) would evidently change more drastically in  $Z_0$  from that of a solid ground plane.

Although the holes of the perforations are squares in Fig. 1, actually only the areas of the holes and parallel plate overlaps are needed in (2). This means that with the area accurately calculated, the periodic holes can be of any shape regular shapes, e.g., rectangles or circles. Formula (2) assumes that the ground plane is infinite in extend.

Because of the perforations on the ground plane, a numerical computation has to include the ground plane and therefore the ground plane has to be finite and not infinite.

In LTCC, the ground plane is finite. This means that (2), of the infinite ground plane, can be used as quick estimate on the properties of the microstrip line, but the final properties (with errors < 2.5%) may still have to be computed numerically or measured in hardware.

Formula (2) is accurate and simple with only one point-matched parameter, the power  $n$ . It therefore shows the convenience of synthetic asymptote in derivation of CAD formulas.

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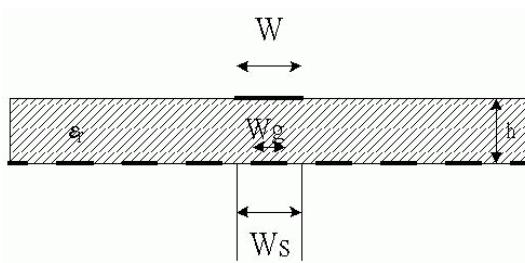


Fig. 1. The cross section view of the microstrip line with perforated ground plane (grid), and the top view of the just 4 grid cells (width:  $W_s$  each) and the strip (width:  $W$ ) above.

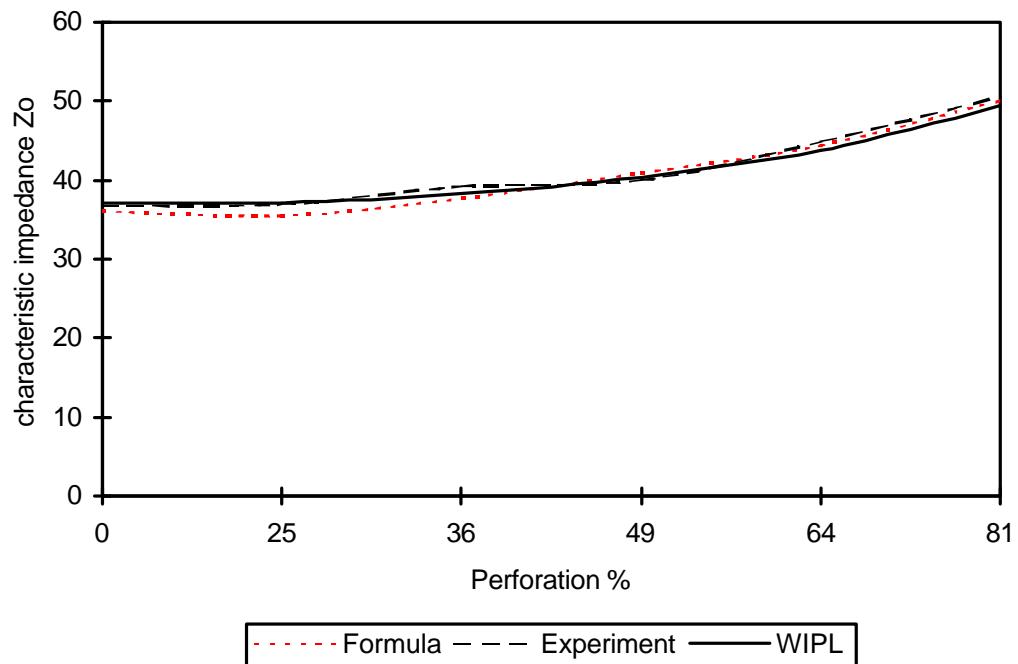
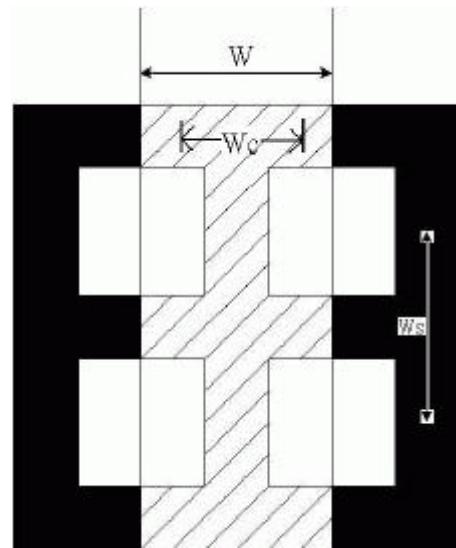


Fig. 2. The characteristic impedance of the line by measurements (at 50 MHz), WIPL computation (at 350 MHz) and by formulas (1) and (2).